**ATMAM Unit 3 – Test 3 – 2017**

**Name**:_____

**Calculator Free Section (No notes or calculators. Formula sheet provided.)**

**Time allowed – 25 minutes** **Marks: 24**

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**Question 1** **[2, 2, 3, 2, 1 marks]**

a) If $f(x)=\dfrac{\sin(2\pi x)}{g(x)}$ and $g(x)\neq 0$, find $f'(x)$

b) Differentiate $y=4x^2\cos(x^3)$

c) Find $\dfrac{d}{dx}(\sin(5-4x))$ and hence find $\int 12\cos(5-4x)dx$

d) If $f'(x) = 2\cos(5x)$ , find $f(x)$

e) $\frac{d}{dx}\left(\int_2^x \tan\theta \; d\theta\right)$

**Question 2 [2 marks]**

Janine drives to work each morning and passes through three traffic intersections with traffic light. The number, X, of traffic lights that are red when Janine is driving to work is a random variable with probability distribution given by:

| *x* | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X = *x*) | 0.1 | 0.2 | 0.3 | 0.4 |

Janine drives to work on two consecutive days. What is the probability that the number of traffic lights that are red is the same on both days?

**Question 3 [3 marks]**

The table below describes the probability distribution for a discrete random variable X.

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X = x)$ | $0.4p^2$ | 0.1 | 0.1 | $1 - 0.6p$ |

Find the value of *p*

**Question 4 [3 marks]**

It has been found that $\frac{1}{5}$ of frozen Coke machines at McJack's restaurants work at any one time. On one afternoon Kyln goes to 4 different McJack's shops. Find the probability that all of them have operational Coke machines?

**Question 5 [4 marks]**

For each situation described below, classify using one of the following phrases:
Bernoulli trial; Binomial Distribution; Uniform Distribution; None
Give a brief explanation for your answer

| **Situation** | **Phrase** | **Reason** |
|---|---|---|
| Prior to an election, a voter is asked whether he will vote for the Labour candidate | | |
| You take a survey of 50 traffic lights in a certain city, at 3 p.m., recording whether the light was red, green, or yellow at that time. | | |

**Question 6 [2 marks]**

Chris has been offered a position selling cars. He will be paid a retainer of $100 per week and a commission of $300 for each car he sells. The table below shows the probability of selling a specific number of cars each week.

| *x* | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| P(X = *x*) | 0.2 | 0.45 | 0.3 | 0.04 | 0.01 |

Calculate Chris's expected weekly pay

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**Calculator Assumed Section (1 A4 page of notes allowed. Formula sheet provided.)**

**Time allowed – 30 minutes** **Marks: 31**

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**Question 7 [4 marks]**

Find the equation of the tangent to the curve with equation $y = 3\sin(2x) - \cos(2x)$ , at the point where $x = \frac{\pi}{4}$

**Question 8 [1, 1, 2, 2 marks]**

Left-handed people make up 9½ % of the population. Four people are randomly selected.

a) Define the distribution

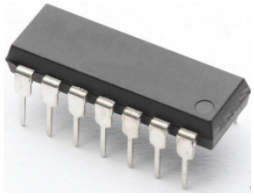
What is the probability that in a randomly selected group of four people:

b) There are exactly 3 right-handed people?

c) There are more left-handed than right-handed people?

d) They are all left-handed, given that there are more left-handed people than right-handed people in the group?

**Question 9 [1, 4, 2 marks]**



It is known that 5% of a batch of computer chips are defective. A sample of twenty chips is randomly selected from this batch.

a) Define the distribution

b) Determine the probability that there:

(i) are no more than 2 defective chips in this sample.

(ii) is at least one defective chip in this sample.

(iii) is no more than 2 defective chips in this sample, if it is known that there is at least 1 defective chip in this sample.

c) Determine the expected number of defective chips in a sample of 1000 chips and its associated standard deviation.

**Question 10 [1, 1, 2 marks]**

The arm of a pendulum swings between its 2 extreme points A and B. Its horizontal displacement $x$ cm from the centre of the swing at time $t$ seconds after it starts swinging is given by $x(t) = 19\sin 4\pi t$

a) State the velocity function $v(t)$

b) What is the maximum speed of the pendulum?

c) Solve for the first 2 times after it starts swinging at which the pendulum reaches its maximum speed

**11 [3 marks]**

For the discrete probability distribution shown below, find the mean and variance. (Note: show working; do not use the STAT menu on your ClassPad)

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **P(X = $x$)** | 0.1 | 0.1 | 0.1 | $a$ | $a$ |

**Question 11 [3, 2, 2 marks]**

A particle moves along a straight line so that its acceleration $a$ in m/s$^2$ at time $t$ seconds is given by:

$$a = -\frac{3\pi^2}{4}\cos\left(\frac{\pi t}{2}\right)$$

Initial velocity is 0 m/s$^2$. Initial displacement is 3 metres to the right of the origin.

Determine:

a) The maximum velocity of the particle, and the time at which this first occurs. *(Show some reasoning for full marks)*

b) An expression for the displacement of the particle at time $t$

c) The total distance travelled by the particle before returning to its initial position.